

हमारा विश्वास... हर एक विद्यार्थी है खास

**JEE  
MAIN  
Sept.  
2020**

**QUESTION PAPER WITH SOLUTION**

**MATHEMATICS \_ 3 Sep. \_ SHIFT - 2**



**MOTION™**

H.O. : 394, Rajeev Gandhi Nagar, Kota  
www.motion.ac.in | ✉: info@motion.ac.in

हमारा विश्वास... हर एक विद्यार्थी है खास

**Q.1** If  $x^3dy + xy dx = x^2dy + 2y dx$ ;  $y(2) = e$  and  $x > 1$ , then  $y(4)$  is equal to:  
यदि  $x^3dy + xy dx = x^2dy + 2y dx$ ;  $y(2) = e$  तथा  $x > 1$  है, तब  $y(4)$  बराबर है-

- (1)  $\frac{\sqrt{e}}{2}$                       (2)  $\frac{3}{2}\sqrt{e}$                       (3)  $\frac{1}{2} + \sqrt{e}$                       (4)  $\frac{3}{2} + \sqrt{e}$

**Sol.**

$$(x^3 - x^2)dy = (2 - x) y dx$$

$$\int \frac{dy}{y} = \int \frac{2 - x}{x^2(x - 1)} dx$$

$$\int \frac{dy}{y} = -\int \frac{x - 1 - 1}{x^2(x - 1)} dx$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x^2} = \int \frac{x^2 - 1 - x^2}{x^2(x - 1)}$$

$$= \frac{1}{x} - \int \frac{x + 1}{x^2} dx + \int \frac{dx}{x - 1}$$

$$\ln|y| = \frac{2}{x} - \ln|x| + \ln|x - 1| + c$$

$$x = 2, y = e$$

$$1 = 1 - \ln 2 + c \Rightarrow c = \ln 2$$

$$\ln|y| = \frac{2}{x} - \ln|x| + \ln|x - 1| + \ln 2$$

$$\text{put } x = 4$$

$$\ln|y| = \frac{1}{2} - 2\ln 2 + \ln 3 + \ln 2$$

$$\ln y = \ln\left(\frac{3}{2}\right) + \frac{1}{2}$$

$$y = \frac{3}{2} \cdot e^{\frac{1}{2}} = \frac{3}{2}\sqrt{e}$$

**CRASH COURSE**  
**FOR JEE ADVANCED 2020**

FREE Online Lectures Available on  YouTube

Go Premium at ₹ 1100

- ◆ Doubt Support ◆ Advanced Level Test Access
- ◆ Live Test Paper Discussion ◆ Final Revision Exercises

Start Date: **07 Sept. 2020**

**Q.2** Let A be a  $3 \times 3$  matrix such that  $\text{adj } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$  and  $B = \text{adj}(\text{adj } A)$ .

If  $|A| = \lambda$  and  $|(B^{-1})^T| = \mu$ , then the ordered pair,  $(|\lambda|, \mu)$  is equal to:

माना A एक  $3 \times 3$  का आव्यूह इस प्रकार है कि  $\text{adj } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$  तथा  $B = \text{adj}(\text{adj } A)$  है।

यदि  $|A| = \lambda$  तथा  $|(B^{-1})^T| = \mu$  है, तब क्रमित युग्म  $(|\lambda|, \mu)$  बराबर है—

- (1)  $\left(9, \frac{1}{81}\right)$       (2)  $\left(9, \frac{1}{9}\right)$       (3)  $\left(3, \frac{1}{81}\right)$       (4)  $(3, 81)$

**Sol. 3**

$$\text{adj } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix} \Rightarrow |\text{adj } A| = 9$$

$$\Rightarrow |A|^2 = 9 \Rightarrow |A| = 3 = |\lambda|$$

$$B = \text{adj}(\text{adj } A) = |A| \cdot A = 3A$$

$$|(B^T)^{-1}| = \frac{1}{|B^T|} = \frac{1}{|B|} = \frac{1}{|3A|} = \frac{1}{27 \times 3} = \frac{1}{81} = \mu$$

$$|\lambda|, \mu = \left(3, \frac{1}{81}\right)$$

**CRASH COURSE**  
FOR JEE ADVANCED 2020

FREE Online Lectures Available on YouTube

Go Premium at ₹ 1100

◆ Doubt Support ◆ Advanced Level Test Access  
◆ Live Test Paper Discussion ◆ Final Revision Exercises

Start Date: **07 Sept. 2020**

हमारा विश्वास... हर एक विद्यार्थी है खास

**Q.3** Let  $a, b, c \in \mathbb{R}$  be such that  $a^2 + b^2 + c^2 = 1$ , If  $a \cos \theta = b \cos\left(\theta + \frac{2\pi}{3}\right) = c \cos\left(\theta + \frac{4\pi}{3}\right)$ , where  $\theta = \frac{\pi}{9}$ , then the angle between the vectors  $a\hat{i} + b\hat{j} + c\hat{k}$  and  $b\hat{i} + c\hat{j} + a\hat{k}$  is

माना  $a, b, c \in \mathbb{R}$  इस प्रकार है, कि  $a^2 + b^2 + c^2 = 1$  है यदि  $a \cos \theta = b \cos\left(\theta + \frac{2\pi}{3}\right) = c \cos\left(\theta + \frac{4\pi}{3}\right)$  है जबकि

$\theta = \frac{\pi}{9}$  है, तब सदिशों  $a\hat{i} + b\hat{j} + c\hat{k}$  तथा  $b\hat{i} + c\hat{j} + a\hat{k}$  के मध्य कोण है-

- (1)  $\frac{\pi}{2}$                       (2)  $\frac{2\pi}{3}$                       (3)  $\frac{\pi}{9}$                       (4) 0

**Sol. 1**

$$\cos \alpha = \frac{ab + bc + ca}{a^2 + b^2 + c^2}$$

$$a \cos \theta = b \cos\left(\theta + \frac{2\pi}{3}\right) = c \cos\left(\theta + \frac{4\pi}{3}\right) = \lambda$$

$$\frac{1}{a} = \frac{\cos \theta}{\lambda}, \frac{1}{b} = \frac{\cos\left(\theta + 2\frac{\pi}{3}\right)}{\lambda}, \frac{1}{c} = \frac{\cos\left(\theta + \frac{4\pi}{3}\right)}{\lambda}$$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{\lambda} \left[ \cos \theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right) \right]$$

$$= \frac{1}{\lambda} \frac{\sin\left[3\left(\frac{\pi}{3}\right)\right]}{\sin\left(\frac{\pi}{3}\right)} \cdot \cos\left[\frac{\theta + \theta + \frac{4\pi}{3}}{2}\right]$$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$$

$$\sum ab = 0$$

$$\cos \alpha = 0$$

$$\alpha = \frac{\pi}{2}$$

**CRASH COURSE**  
FOR JEE ADVANCED 2020

FREE Online Lectures Available on YouTube

Go Premium at ₹ 1100

◆ Doubt Support ◆ Advanced Level Test Access  
◆ Live Test Paper Discussion ◆ Final Revision Exercises

Start Date: **07 Sept. 2020**

**Q.4** Suppose  $f(x)$  is a polynomial of degree four, having critical points at  $-1, 0, 1$ . If  $T = \{x \in \mathbb{R} \mid f(x) = f(0)\}$ , then the sum of squares of all the elements of  $T$  is:

माना  $f(x)$  चार घात का एक बहुपद है, जिसके क्रान्तिक बिन्दु  $-1, 0, 1$  हैं। यदि  $T = \{x \in \mathbb{R} \mid f(x) = f(0)\}$  है, तब  $T$  के सभी अवयवों के वर्गों का योगफल है—

- (1) 6 (2) 2 (3) 8 (4) 4

**Sol.**

**4**  
 $f'(x) = k(x+1)x(x-1)$   
 $f'(x) = k[x^3 - x]$   
 Integrating both sides

$$f(x) = k \left[ \frac{x^4}{4} - \frac{x^2}{2} \right] + C$$

$$f(0) = C$$

$$f(x) = f(0) \Rightarrow k \left( \frac{x^4}{4} - \frac{x^2}{2} \right) + C = C$$

$$\Rightarrow k \frac{x^2}{4} (x^2 - 2) = 0$$

$$\Rightarrow x = 0, \pm \sqrt{2}$$

$$\text{sum of all of squares of elements} = 0^2 + (\sqrt{2})^2 + (-\sqrt{2})^2 = 4$$

**Q.5** If the value of the integral  $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}} dx$  is  $\frac{k}{6}$ , then  $k$  is equal to:

यदि समाकलन  $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}} dx$  का मान  $\frac{k}{6}$  है, तब  $k$  बराबर है—

- (1)  $2\sqrt{3} + \pi$  (2)  $3\sqrt{2} + \pi$  (3)  $3\sqrt{2} - \pi$  (4)  $2\sqrt{3} - \pi$

**Sol.**

**4**  
 $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}} dx$

$$x = \sin \theta$$

$$\int_0^{\pi/6} \frac{\sin^2 \theta}{\cos^3 \theta} \cdot \cos \theta d\theta$$

$$\int_0^{\pi/6} \tan^2 \theta d\theta = [\tan \theta - \theta]_0^{\pi/6}$$

**CRASH COURSE**  
**FOR JEE ADVANCED 2020**

FREE Online Lectures Available on YouTube

Go Premium at ₹ 1100

- ◆ Doubt Support ◆ Advanced Level Test Access
- ◆ Live Test Paper Discussion ◆ Final Revision Exercises

Start Date: **07 Sept. 2020**

हमारा विश्वास... हर एक विद्यार्थी है खास

$$\Rightarrow \left( \frac{1}{\sqrt{3}} - \frac{\pi}{6} \right) = \frac{k}{6}$$

$$\frac{2\sqrt{3} - \pi}{6} = \frac{k}{6}$$

$$k = 2\sqrt{3} - \pi$$

**Q.6** If the term independent of  $x$  in the expansion of  $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$  is  $k$ , then  $18k$  is equal to:

यदि  $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$  के प्रसार में  $x$  से स्वतंत्र पद  $k$  है, तब  $18k$  बराबर है—

(1) 5

(2) 9

(3) 7

(4) 11

**Sol. 3**

$$T_{r+1} = {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(\frac{-1}{3x}\right)^r$$

$$= {}^9C_r \frac{3^{9-2r}}{2^{9-r}} (-1)^r \cdot x^{18-3r}$$

$$18 - 3r = 0$$

$$\Rightarrow r=6$$

$$= {}^9C_r \left(\frac{3^{-3}}{2^3}\right) = k$$

$$= \frac{7}{18} = k \Rightarrow 18k = 7$$

**7.** If a  $\triangle ABC$  has vertices  $A(-1,7)$ ,  $B(-7,1)$  and  $C(5,-5)$ , then its orthocentre has coordinates:

यदि एक  $\triangle ABC$  के शीर्ष  $A(-1,7)$ ,  $B(-7,1)$  तथा  $C(5,-5)$  है, तब इसके लम्बकेन्द्र के निर्देशांक है—

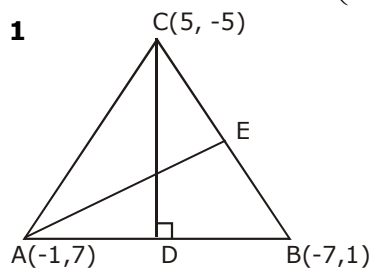
(1)  $(-3,3)$

(2)  $\left(-\frac{3}{5}, \frac{3}{5}\right)$

(3)  $\left(\frac{3}{5}, -\frac{3}{5}\right)$

(4)  $(3,-3)$

**Sol. 1**



**CRASH COURSE**  
FOR JEE ADVANCED 2020

FREE Online Lectures Available on YouTube

Go Premium at ₹ 1100

◆ Doubt Support ◆ Advanced Level Test Access  
◆ Live Test Paper Discussion ◆ Final Revision Exercises

Start Date: **07 Sept. 2020**

equation of CD  
 $y + 5 = -1(x - 5)$   
 $x + y = 0$  .....(1)  
 equation of AE  
 $y - 7 = 2(x + 1)$   
 $2x - y = -9$  .....(2)  
 from (1) & (2)  
 $x = -3, y = 3$   
 Othocentre = (-3, 3)

**Q.8.** Let  $e_1$  and  $e_2$  be the eccentricities of the ellipse,  $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$  ( $b < 5$ ) and the hyperbola,  $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$  respectively satisfying  $e_1 e_2 = 1$ . If  $\alpha$  and  $\beta$  are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair  $(\alpha, \beta)$  is equal to:

माना दीर्घवत्त  $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$  ( $b < 5$ ) तथा अतिपरवलय  $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$  की उत्क्रेन्द्रताए क्रमशः  $e_1$  तथा  $e_2$  है जो  $e_1 e_2 = 1$  को सन्तुष्ट करती है। यदि दीर्घवत्त तथा अतिपरवलय की नाभियों के बीच की दूरियां क्रमशः  $\alpha$  तथा  $\beta$  है तब क्रमित युग्म  $(\alpha, \beta)$  बराबर है—

- (1) (8,12)                      (2)  $\left(\frac{24}{5}, 10\right)$                       (3)  $\left(\frac{20}{3}, 12\right)$                       (4) (8,10)

**Sol. 4**

$$\left. \begin{aligned} \alpha &= 10e_1 \\ \beta &= 8e_2 \end{aligned} \right\} \quad \left. \begin{aligned} b^2 &= 25(1 - e_1^2) \\ b^2 &= 16(e_2^2 - 1) \end{aligned} \right\}$$

$$(e_1 e_2)^2 = 1$$

$$\left(1 - \frac{b^2}{25}\right) \left(1 + \frac{b^2}{16}\right) = 1$$

$$\Rightarrow 1 + \frac{b^2}{25} - \frac{b^2}{25} - \frac{b^4}{400} = 1$$

$$\Rightarrow \frac{9}{16.25} b^2 = \frac{b^4}{400} \Rightarrow b^2 = 9$$

$$\left. \begin{aligned} e_1 &= \frac{4}{5} \\ e_2 &= \frac{5}{4} \end{aligned} \right\} = \left. \begin{aligned} \alpha &= 2ae_1 = 10 \times \frac{4}{5} = 8 \\ \beta &= 2ae_2 = 8 \times \frac{5}{4} = 10 \end{aligned} \right\} = (\alpha, \beta) = (8, 10)$$

हमारा विश्वास... हर एक विद्यार्थी है खास

**Q.9** If  $z_1, z_2$  are complex numbers such that  $\operatorname{Re}(z_1) = |z_1 - 1|$ ,  $\operatorname{Re}(z_2) = |z_2 - 1|$  and  $\arg(z_1 - z_2) = \frac{\pi}{6}$ , then  $\operatorname{Im}(z_1 + z_2)$  is equal to:

यदि  $z_1, z_2$  सम्मिश्र संख्या इस प्रकार है कि  $\operatorname{Re}(z_1) = |z_1 - 1|$ ,  $\operatorname{Re}(z_2) = |z_2 - 1|$  तथा  $\arg(z_1 - z_2) = \frac{\pi}{6}$  है, तब  $\operatorname{Im}(z_1 + z_2)$  बराबर है—

- (1)  $2\sqrt{3}$                       (2)  $\frac{2}{\sqrt{3}}$                       (3)  $\frac{1}{\sqrt{3}}$                       (4)  $\frac{\sqrt{3}}{2}$

**Sol. 1**

$$z_1 = x_1 + iy_1, z_2 = x_2 + iy_2$$

$$x_1^2 = (x_1 - 1)^2 + y_1^2 \quad \dots(1)$$

$$\Rightarrow y_1^2 - 2x_1 + 1 = 0$$

$$x_2^2 = (x_2 - 1)^2 + y_2^2 \quad \dots(2)$$

$$y_2^2 - 2x_2 - 1 = 0$$

from equation (1) - (2)

$$(y_1^2 - y_2^2) + 2(x_2 - x_1) = 0$$

$$(y_1 + y_2)(y_1 - y_2) = 2(x_1 - x_2)$$

$$y_1 + y_2 = 2 \left( \frac{x_1 - x_2}{y_1 - y_2} \right)$$

$$\arg(z_1 - z_2) = \frac{\pi}{6}$$

$$\tan^{-1} \left( \frac{y_1 - y_2}{x_1 - x_2} \right) = \frac{\pi}{6}$$

$$\Rightarrow \frac{y_1 - y_2}{x_1 - x_2} = \frac{1}{\sqrt{3}}$$

$$\therefore y_1 + y_2 = 2\sqrt{3}$$

**Q.10** The set of all real values of  $\lambda$  for which the quadratic equations,  $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$  always have exactly one root in the interval (0,1) is:

$\lambda$  के सभी वास्तविक मानों का समुच्च, जिसके लिये द्विघात समीकरण  $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$  अंतराल (0,1) में हमेशा ठीक एक मूल रखती है, होगा—

- (1) (-3, -1)                      (2) (2, 4]                      (3) (1, 3]                      (4) (0, 2)

**CRASH COURSE**  
FOR JEE ADVANCED 2020

FREE Online Lectures Available on YouTube

Go Premium at ₹ 1100

- ◆ Doubt Support
- ◆ Advanced Level Test Access
- ◆ Live Test Paper Discussion
- ◆ Final Revision Exercises

Start Date: **07 Sept. 2020**



**Sol. 3**

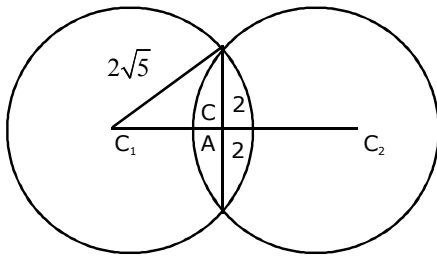
$$\begin{aligned} f(0) f(1) &\leq 0 \\ \Rightarrow (2) [\lambda^2 - 4\lambda + 3] &\leq 0 \\ (\lambda - 1) (\lambda - 3) &\leq 0 \\ \Rightarrow \lambda &\in [1, 3] \\ \text{at } \lambda &= 1 \\ 2x^2 - 4x + 2 &= 0 \\ \Rightarrow (x - 1)^2 &= 0 \\ x &= 1, 1 \\ \therefore \lambda &\in (1, 3] \end{aligned}$$

**Q.11** Let the latus ractum of the parabola  $y^2=4x$  be the common chord to the circles  $C_1$  and  $C_2$  each of them having radius  $2\sqrt{5}$ . Then, the distance between the centres of the circles  $C_1$  and  $C_2$  is:

माना मखलय  $y^2=4x$  का नाभिलम्ब वत्तों  $C_1$  तथा  $C_2$  की उभयनिष्ठ जीवा है इनमें से प्रत्येक की त्रिज्या  $2\sqrt{5}$  है, तब वत्तों  $C_1$  तथा  $C_2$  के केन्द्रों के बीच की दूरी है-

- (1) 8                                      (2)  $8\sqrt{5}$                                       (3)  $4\sqrt{5}$                                       (4) 12

**Sol. 1**



$$\begin{aligned} C_1C_2 &= 2C_1A \\ (C_1A)^2 + 4 &= (2\sqrt{5})^2 \\ C_1A &= 4 \\ C_1C_2 &= 8 \end{aligned}$$

**Q.12** The plane which bisects the line joining the points  $(4,-2,3)$  and  $(2,4,-1)$  at right angles also passes through the point:

वह समतल जो बिन्दुओं  $(4,-2,3)$  तथा  $(2,4,-1)$  को मिलाने वाली रेखा को लम्ब समद्विभाजित करता है, निम्न में से किस बिन्दु से होकर गुजरता है-

- (1)  $(0,-1,1)$                                       (2)  $(4,0,1)$                                       (3)  $(4,0,-1)$                                       (4)  $(0,1,-1)$

**Sol. 3**

$$\begin{aligned} &A \text{---} (3, 1, 1) \text{---} B \\ &(y, -2, -3) \quad (3, 1, 1) \quad (2, 4, -1) \\ a &= 2, b = -6 \\ c &= 4 \\ &\text{equation of plane} \end{aligned}$$

**CRASH COURSE**  
FOR JEE ADVANCED 2020

FREE Online Lectures Available on YouTube

Go Premium at ₹ 1100

- ◆ Doubt Support
- ◆ Advanced Level Test Access
- ◆ Live Test Paper Discussion
- ◆ Final Revision Exercises

Start Date: **07 Sept. 2020**

हमारा विश्वास... हर एक विद्यार्थी है खास

$$2(x - 3) + (-6)(y - 1) + 4(z - 1) = 0$$

$$\Rightarrow 2x - 6y + 4z = 4$$

passes through (4, 0, -1)

**Q.13**  $\lim_{x \rightarrow a} \frac{(a+2x)^{\frac{1}{3}} - (3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}} - (4x)^{\frac{1}{3}}}$  ( $a \neq 0$ ) is equal to :

$\lim_{x \rightarrow a} \frac{(a+2x)^{\frac{1}{3}} - (3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}} - (4x)^{\frac{1}{3}}}$  ( $a \neq 0$ ) बराबर हैं—

(1)  $\left(\frac{2}{9}\right)^{\frac{4}{3}}$

(2)  $\left(\frac{2}{3}\right)^{\frac{4}{3}}$

(3)  $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$

(4)  $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

**Sol. 3**

Apply L-H Rule

$$\lim_{x \rightarrow a} \frac{\frac{2}{3}(a+2x)^{-\frac{2}{3}} - 3^{\frac{1}{3}} \cdot \frac{1}{3} x^{-\frac{2}{3}}}{\frac{1}{3}(3a+x)^{-\frac{2}{3}} - 4^{\frac{1}{3}} \cdot \frac{1}{3} x^{-\frac{2}{3}}}$$

$$\Rightarrow \frac{\frac{2}{3}(3a)^{-\frac{2}{3}} - \frac{1}{3} \cdot \left(a^{-\frac{2}{3}}\right)}{\frac{1}{3}(4a)^{-\frac{2}{3}} - \frac{1}{3} \cdot 4^{\frac{1}{3}} \left(a^{-\frac{2}{3}}\right)}$$

$$= \frac{2}{3} \cdot \left(\frac{2}{9}\right)^{\frac{1}{3}}$$

**CRASH COURSE**  
FOR JEE ADVANCED 2020

FREE Online Lectures Available on YouTube

Go Premium at ₹ 1100

◆ Doubt Support ◆ Advanced Level Test Access  
◆ Live Test Paper Discussion ◆ Final Revision Exercises

Start Date: **07 Sept. 2020**

**Q.14** Let  $x_i (1 \leq i \leq 10)$  be ten observations of a random variable  $X$ . If  $\sum_{i=1}^{10} (x_i - p) = 3$  and  $\sum_{i=1}^{10} (x_i - p)^2 = 9$  where  $0 \neq p \in \mathbb{R}$ , then the standard deviation of these observations is :

माना यादच्छिक चर  $x$  के दस प्रेक्षण  $x_i (1 \leq i \leq 10)$  हैं। यदि  $\sum_{i=1}^{10} (x_i - p) = 3$  तथा  $\sum_{i=1}^{10} (x_i - p)^2 = 9$  है, जहाँ  $0 \neq p \in \mathbb{R}$  हैं तब इन प्रेक्षणों का मानक विचलन है।

- (1)  $\frac{7}{10}$                       (2)  $\frac{9}{10}$                       (3)  $\sqrt{\frac{3}{5}}$                       (4)  $\frac{4}{5}$

**Sol. 2**

Standard deviation is free from shifting of origin

$$S.D = \sqrt{\text{variance}}$$

$$= \sqrt{\frac{9}{10} - \left(\frac{3}{10}\right)^2}$$

$$= \sqrt{\frac{9}{10} - \frac{9}{100}}$$

$$= \sqrt{\frac{81}{100}} = \frac{9}{10}$$

**Q.15** The probability that a randomly chosen 5-digit number is made from exactly two digits is :  
यादच्छिक रूप से चुनी गई 5-अंको की संख्या के मात्र दो अंको से बनाई गई होने की प्रायिकता है—

- (1)  $\frac{134}{10^4}$                       (2)  $\frac{121}{10^4}$                       (3)  $\frac{135}{10^4}$                       (4)  $\frac{150}{10^4}$

**Sol. 3**

$$\begin{aligned} \text{Total case} &= 9(10^4) \\ \text{fav. case} &= {}^9C_2 (2^5 - 2) + {}^9C_1 (2^4 - 1) \\ &= 1080 + 135 = 1215 \end{aligned}$$

$$\text{Prob} = \frac{1215}{9 \times 10^4} = \frac{135}{10^4}$$

हमारा विश्वास... हर एक विद्यार्थी है खास

**Q.16** If  $\int \sin^{-1} \left( \sqrt{\frac{x}{1+x}} \right) dx = A(x) \tan^{-1}(\sqrt{x}) + B(x) + C$ , where C is a constant of integration, then the ordered pair (A(x), B(x)) can be:

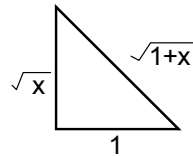
यदि  $\int \sin^{-1} \left( \sqrt{\frac{x}{1+x}} \right) dx = A(x) \tan^{-1}(\sqrt{x}) + B(x) + C$ , है, जहाँ C समाकलन का एक निपतांक है, तब क्रमित युग्म

(A(x), B(x)) हो सकते हैं, होंगे –

- (1)  $(x+1, -\sqrt{x})$       (2)  $(x-1, -\sqrt{x})$       (3)  $(x+1, \sqrt{x})$       (4)  $(x-1, \sqrt{x})$

**Sol. 1**

$$\int \sin^{-1} \sqrt{\frac{x}{1+x}} dx$$



$$\int \tan^{-1} \sqrt{x} \cdot \frac{1}{2\sqrt{x}} dx$$

$$(\tan^{-1} \sqrt{x}) \cdot x - \int \frac{x}{1+x} \cdot \frac{1}{2\sqrt{x}} dx$$

put  $x = t^2 \Rightarrow dx = 2t dt$

$$= x \tan^{-1} \sqrt{x} - \int \frac{(t^2)(2t dt)}{(1+t^2)(2t)}$$

$$= x \tan^{-1} \sqrt{x} - t + \tan^{-1} t + c$$

$$= x \tan^{-1} \sqrt{x} - \sqrt{x} + \tan^{-1} \sqrt{x} + c$$

$$A(x) = x + 1, B(x) = -\sqrt{x}$$

**Q.17** If the sum of the series  $20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \dots$  upto  $n^{\text{th}}$  term is 488 and the  $n^{\text{th}}$  term is negative, then:

- (1)  $n=60$       (2)  $n=41$       (3)  $n^{\text{th}}$  term is  $-4$       (4)  $n^{\text{th}}$  term is  $-4\frac{2}{5}$

**CRASH COURSE**  
FOR JEE ADVANCED 2020

FREE Online Lectures Available on YouTube

Go Premium at ₹ 1100

- ◆ Doubt Support ◆ Advanced Level Test Access
- ◆ Live Test Paper Discussion ◆ Final Revision Exercises

Start Date: **07 Sept. 2020**

यदि श्रेणी  $20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \dots$  का  $n^{\text{th}}$  पद तक का योगफल 488 है तथा  $n^{\text{th}}$  पद ऋणात्मक है तब

(1)  $n=60$

(2)  $n=41$

(3)  $n^{\text{th}}$  पद  $-4$  है

(4)  $n^{\text{th}}$  पद  $-4\frac{2}{5}$  है

**Sol. 3**

$$20 + \frac{98}{5} + \frac{96}{5} + \dots$$

$$S_n = 488$$

$$\Rightarrow \frac{n}{2} \left[ 2 \times 20 + (n-1) \left( \frac{-2}{5} \right) \right] = 488$$

$$\Rightarrow 20n - \frac{n^2}{5} + \frac{n}{5} = 488$$

$$\Rightarrow 100n - n^2 + n = 2440$$

$$= n^2 - 101n + 2440 = 0$$

$$\Rightarrow n = 61 \text{ or } 40$$

$$\text{for } n = 40, T_n = 20 + 39 \left( \frac{-2}{5} \right) = +ve$$

$$n = 61, T_n = 20 + 60 \left( \frac{-2}{5} \right) = 20 - 24 = -4$$

**Q.18** Let  $p, q, r$  be three statements such that the truth value of  $(p \wedge q) \rightarrow (\sim p \vee r)$  is F. Then the truth values of  $p, q, r$  are respectively :

माना  $p, q, r$  तीन कथन इस प्रकार है कि  $(p \wedge q) \rightarrow (\sim p \vee r)$  का सत्यता मान F है, तब  $p, q, r$  के सत्यता मान क्रमशः है—

(1) F, T, F

(2) T, F, T

(3) T, T, F

(4) T, T, T

**Sol. 3**

$$(p \wedge q) \rightarrow (\sim q \vee r)$$

Possible when

$$p \wedge q \rightarrow T$$

$$\sim q \vee r \rightarrow F$$

$$p \rightarrow T$$

$$q \rightarrow T$$

$$r \rightarrow F$$

$$p \wedge q \Rightarrow T$$

$$\sim q \vee r \rightarrow F \vee F \Rightarrow F$$

$$T \rightarrow F \Rightarrow F$$

**CRASH COURSE**  
FOR JEE ADVANCED 2020

FREE Online Lectures Available on YouTube

Go Premium at ₹ 1100

◆ Doubt Support ◆ Advanced Level Test Access  
◆ Live Test Paper Discussion ◆ Final Revision Exercises

Start Date: **07 Sept. 2020**

हमारा विश्वास... हर एक विद्यार्थी है खास

- Q.19** If the surface area of a cube is increasing at a rate of  $3.6 \text{ cm}^2/\text{sec}$ , retaining its shape; then the rate of change of its volume (in  $\text{cm}^3/\text{sec}$ ), when the length of a side of the cube is  $10\text{cm}$ , is :  
 यदि अपने रूप को बनाये रखते हुये एक घन का प्रष्टीय क्षेत्रफल  $3.6 \text{ cm}^2/\text{sec}$  की एक दर से बड रहा है, तब इसके आयतन के परिवर्तन की दर ( $\text{cm}^3/\text{sec}$  में) जब घन के एक भुजा की लम्बाई  $10\text{cm}$  है, है—  
 (1) 9 (2) 10 (3) 18 (4) 20

**Sol. 1**

$$A = 6a^2$$

$a \rightarrow$  side of cube

$$\frac{dA}{dt} = 6 \left( 2a \frac{da}{dt} \right) \Rightarrow 3.6 = 12 \times 10 \frac{da}{dt} \Rightarrow \frac{da}{dt} = \frac{3}{100}$$

$$v = a^3$$

$$\frac{dV}{dt} = 3a^2 \frac{da}{dt}$$

$$= 3 \times 100 \times \frac{3}{100}$$

$$= 9\text{cm}^3 / \text{sec}$$

- Q.20** Let  $R_1$  and  $R_2$  be two relations defined as follows:

$$R_1 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \in \mathbb{Q}\} \text{ and}$$

$$R_2 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \notin \mathbb{Q}\}, \text{ where } \mathbb{Q} \text{ is the set of all rational numbers. Then :}$$

- (1)  $R_1$  is transitive but  $R_2$  is not transitive  
 (2)  $R_1$  and  $R_2$  are both transitive  
 (3)  $R_2$  is transitive but  $R_1$  is not transitive  
 (4) Neither  $R_1$  nor  $R_2$  is transitive

माना  $R_1$  तथा  $R_2$  दो सम्बन्ध निम्न रूप में परिभाषित है

$$R_1 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \in \mathbb{Q}\} \text{ तथा}$$

$$R_2 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \notin \mathbb{Q}\}, \text{ जहाँ } \mathbb{Q} \text{ सभी परिमेय संख्याओं का समुच्च है। तब}$$

- (1)  $R_1$  संक्रामक है लेकिन  $R_2$  संक्रामक नहीं है  
 (2)  $R_1$  तथा  $R_2$  दोनों संक्रामक है  
 (3)  $R_2$  संक्रामक है लेकिन  $R_1$  संक्रामक नहीं है  
 (4) ना तो  $R_1$  ना  $R_2$  संक्रामक है

**Sol. 4**

for  $R_1$

$$\text{Let } a = 1 + \sqrt{2}, b = 1 - \sqrt{2}, c = \frac{1}{8^{\frac{1}{4}}}$$

$$aR_1b \quad a^2 + b^2 = (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 6 \in \mathbb{Q}$$

$$bR_1c \quad b^2 + c^2 = (1 - \sqrt{2})^2 + \left( \frac{1}{8^{\frac{1}{4}}} \right)^2 = 3 \in \mathbb{Q}$$

**CRASH COURSE**  
**FOR JEE ADVANCED 2020**

FREE Online Lectures Available on  YouTube

Go Premium at ₹ 1100

- ◆ Doubt Support ◆ Advanced Level Test Access
- ◆ Live Test Paper Discussion ◆ Final Revision Exercises

Start Date: **07 Sept. 2020**

$$aR_1c \Rightarrow a^2+c^2 = (1+\sqrt{2})+(8^{1/4})^2 = 3+4\sqrt{2} \notin Q$$

$R_1$  is not transitive

$R_2$

$$\text{let } a = 1 + \sqrt{2}, b = \sqrt{2}, c = 1 - \sqrt{2}$$

$$aR_2b \quad a^2 + b^2 = 5 + 2\sqrt{2} \notin Q$$

$$bR_2c \quad b^2 + c^2 = 5 - 2\sqrt{2} \notin Q$$

$$aR_2c \quad a^2 + c^2 = 6 \in Q$$

$R_2$  is not transitive

**Q.21** If  $m$  arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4<sup>th</sup> A.M. is equal to 2<sup>nd</sup> G.M., then  $m$  is equal to \_\_\_\_

यदि 3 तथा 243 के बीच  $m$  समान्तर माध्य तथा तीन गुणोत्तर माध्य इस प्रकार डाले गये हैं कि 4<sup>th</sup> समान्तर माध्य 2<sup>nd</sup> गुणोत्तर माध्य दूसरे गुणोत्तर माध्य के बराबर है तब  $m$  बराबर है—

**Sol.** **39**

$$3, \dots, 243$$

m A.M.

$$3, \dots, 243$$

3 G.M

$$d = \frac{b-a}{n+1} = \frac{243-3}{m+1} = \frac{240}{m+1}$$

$$243 = 3(r)^4$$

$$4^{\text{th}} \text{ A.M} = 3 + 4d = 3 + 4\left(\frac{240}{m+1}\right)$$

$$r = 3$$

$$3 + \frac{960}{m+1} = 27$$

$$2^{\text{nd}} \text{ G.M.} = ar^2 = 27$$

$$= \frac{960}{m+1} = 24$$

$$\Rightarrow m = 39$$

**Q.22** Let a plane  $P$  contain two lines  $\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j}), \lambda \in \mathbb{R}$  and  $\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k}), \mu \in \mathbb{R}$ . If  $Q(\alpha, \beta, \gamma)$  is the foot of the perpendicular drawn from the point  $M(1, 0, 1)$  to  $P$ , then  $3(\alpha + \beta + \gamma)$  equals \_\_\_\_

माना एक समतल  $P$  पर दो रेखाएँ  $\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j}), \lambda \in \mathbb{R}$  तथा  $\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k}), \mu \in \mathbb{R}$  स्थित हैं। यदि  $Q(\alpha, \beta, \gamma)$ , बिन्दु  $M(1, 0, 1)$  से  $P$  पर खींचा गया लम्बपाद है, तब  $3(\alpha + \beta + \gamma)$  बराबर है—

**Sol.** **5**

$$\left. \begin{aligned} \vec{r} &= \hat{i} + \lambda(\hat{i} + \hat{j}) \\ \vec{r} &= -\hat{j} + \mu(\hat{j} - \hat{k}) \end{aligned} \right\}$$

**CRASH COURSE**  
FOR JEE ADVANCED 2020

FREE Online Lectures Available on YouTube

Go Premium at ₹ 1100

◆ Doubt Support ◆ Advanced Level Test Access  
◆ Live Test Paper Discussion ◆ Final Revision Exercises

Start Date: **07 Sept. 2020**

हमारा विश्वास... हर एक विद्यार्थी है खास

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= (-1, 1, 1)$$

equation of plane

$$-1(x - 1) + 1(y - 0) + 1(z - 0) = 0$$

$$\Rightarrow x - y - z - 1 = 0$$

foot of  $\perp r$  from  $m(1, 0, 1)$

$$\frac{x-1}{1} = \frac{y-0}{-1} = \frac{z-1}{-1} = -\frac{(1-0-1-1)}{3}$$

$$x-1 = \frac{1}{3} \quad \left| \frac{y}{-1} = \frac{1}{3} \right| = \frac{z-1}{-1} = \frac{1}{3}$$

$$x = \frac{4}{3}, y = \frac{-1}{3}, z = \frac{2}{3}$$

$$\Rightarrow \left. \begin{array}{l} \alpha = \frac{4}{3} \\ \beta = \frac{-1}{3} \\ \gamma = \frac{2}{3} \end{array} \right\}$$

$$\alpha + \beta + \gamma = \frac{4}{3} - \frac{1}{3} + \frac{2}{3} = \frac{5}{3}$$

$$3(\alpha + \beta + \gamma) = 5$$

**Q.23** Let S be the set of all integer solutions,  $(x, y, z)$ , of the system of equations

$$x - 2y + 5z = 0$$

$$-2x + 4y + z = 0$$

$$-7x + 14y + 9z = 0$$

such that  $15 \leq x^2 + y^2 + z^2 \leq 150$ . Then, the number of elements in the set S is equal to \_\_\_\_

माना S, समीकरण निकाय

$$x - 2y + 5z = 0$$

$$-2x + 4y + z = 0$$

$$-7x + 14y + 9z = 0$$

के सभी पूर्णांक हलों  $(x, y, z)$  का समुच्चय इस प्रकार है कि

$15 \leq x^2 + y^2 + z^2 \leq 150$  है तब समुच्चय S में अवयवों की संख्या बराबर है—

**CRASH COURSE**  
FOR JEE ADVANCED 2020

FREE Online Lectures Available on YouTube

Go Premium at ₹ 1100

◆ Doubt Support ◆ Advanced Level Test Access  
◆ Live Test Paper Discussion ◆ Final Revision Exercises

Start Date: **07 Sept. 2020**



**Sol. 8**

$$x - 2y + 5z = 0 \quad \dots(1)$$

$$-2x + 4y + z = 0 \quad \dots(2)$$

$$-7x + 14y + 9z = 0 \quad \dots(3)$$

2.(1) + (2) we get  $z = 0, x = 2y$

$$15 \leq 4y^2 + y^2 \leq 150$$

$$\Rightarrow 3 \leq y^2 \leq 30$$

$$y \in [-\sqrt{30}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{30}]$$

$$y = \pm 2, \pm 3, \pm 4, \pm 5$$

no. of integer's in S is 8

**Q.24** The total number of 3-digit numbers, whose sum of digits is 10, is \_\_\_\_\_

3-अंको की संख्याएँ जिनके अंको का योगफल 10 है की कुल संख्या है-

**Sol. 54**

Let xyz be 3 digit number

$$x + y + z = 10 \text{ where } x \geq 1, y \geq 0, z \geq 0$$

$$\Rightarrow t + y + z = 9$$

$$\left. \begin{array}{l} x - 1 \geq 0 \\ t \geq 0 \end{array} \right\} x - 1 = t$$

$${}^{9+3-1}C_{3-1} = {}^{11}C_2 = 55$$

but for  $t = 9, x = 10$  not possible

$$\text{total numbers} = 55 - 1 = 54$$

**Q.25** If the tangent to the curve,  $y=e^x$  at a point  $(c, e^c)$  and the normal to the parabola,  $y^2=4x$  at the point  $(1,2)$  intersect at the same point on the x-axis, then the value of c is \_\_\_\_\_

यदि वक्र  $y=e^x$  के एक बिन्दु  $(c, e^c)$  पर स्पर्शरेखा तथा परवलय  $y^2=4x$  के बिन्दु  $(1,2)$  पर अभिलंब x-अक्ष पर समान बिन्दु पर प्रतिच्छेद करते हैं, तब c का मान है-

**Sol. 4**

$$\text{Tangent at } (c, e^c) \quad y - e^c = e^c (x - c) \quad \dots(1)$$

$$\text{normal to parabola } y - 2 = -1 (x - 1)$$

$$x + y = 3 \quad \dots(2)$$

$$\text{at x-axis } y = 0 \quad \text{at x-axis } y = 0$$

$$\text{in (1), } x = c - 1 \quad \text{in (2), } x = 3$$

$$c - 1 = 3 \Rightarrow c = 4$$

**CRASH COURSE**  
FOR JEE ADVANCED 2020

FREE Online Lectures Available on YouTube

Go Premium at ₹ 1100

◆ Doubt Support ◆ Advanced Level Test Access  
◆ Live Test Paper Discussion ◆ Final Revision Exercises

Start Date: **07 Sept. 2020**

Admission  
**OPEN**

जब इन्होंने पूरा किया अपना सपना  
तो आप भी पा सकते है लक्ष्य अपना

## JEE MAIN RESULT 2019



**Nitin Gupta**

Marks  
**335**  
13th (2019)

Marks  
**149**  
12th (2018)



**Shiv Modi**

Marks  
**318**  
13th (2019)

Marks  
**153**  
12th (2018)



**Ritik Bansal**

Marks  
**308**  
13th (2019)

Marks  
**218**  
12th (2018)



**Shubham Kumar**

Marks  
**300**  
13th (2019)

Marks  
**153**  
12th (2018)

### KOTA'S PIONEER IN DIGITAL EDUCATION

**1,95,00,000+** viewers | **72,67,900+** viewing hours | **2,11,000+** Subscribers

SERVICES	SILVER	GOLD	PLATINUM
Classroom Lectures (VOD)			
Live interaction	NA		
Doubt Support	NA		
Academic & Technical Support	NA		
Complete access to all content	NA		
Classroom Study Material	NA		
Exercise Sheets	NA		
Recorded Video Solutions	NA		
Online Test Series	NA		
Revision Material	NA		
<b>Upgrade to Regular Classroom program</b>	<b>Chargeable</b>	<b>Chargeable</b>	<b>Free</b>
Physical Classroom	NA	NA	
Computer Based Test	NA	NA	
Student Performance Report	NA	NA	
Workshop & Camp	NA	NA	
Motion Solution Lab- Supervised learning and instant doubt clearance	NA	NA	
Personalised guidance and mentoring	NA	NA	

#### FEE STRUCTURE

CLASS	SILVER	GOLD	PLATINUM
7th/8th	FREE	₹ 12,000	₹ 35,000
9th/10th	FREE	₹ 15,000	₹ 40,000
11th	FREE	₹ 29,999	₹ 49,999
12th	FREE	₹ 39,999	₹ 54,999
12th Pass	FREE	₹ 39,999	₹ 59,999

+ Student Kit will be provided at extra cost to Platinum Student.

- \* **SILVER (Trial)** Only valid 7 DAYS or First 10 Hour's Lectures.
- \*\* **GOLD (Online)** can be converted to regular classroom (Any MOTION Center) by paying difference amount after lockdown.
- \*\*\* **PLATINUM (Online + Regular)** can be converted to regular classroom (Any MOTION Center) without any cost after lockdown.

New Batch Starting from :  
**16 & 23 September 2020**

**Zero Cost EMI Available**

**MOTION™**

H.O. : 394, Rajeev Gandhi Nagar, Kota  
www.motion.ac.in | ✉ : info@motion.ac.in